

Non-linear Graphs

A graph can be used to illustrate the relationship between two variables.

If the graph is not a straight line, it is said to be **non-linear**.

Some relationships may be given by a formula.

Example - Area of a Circle

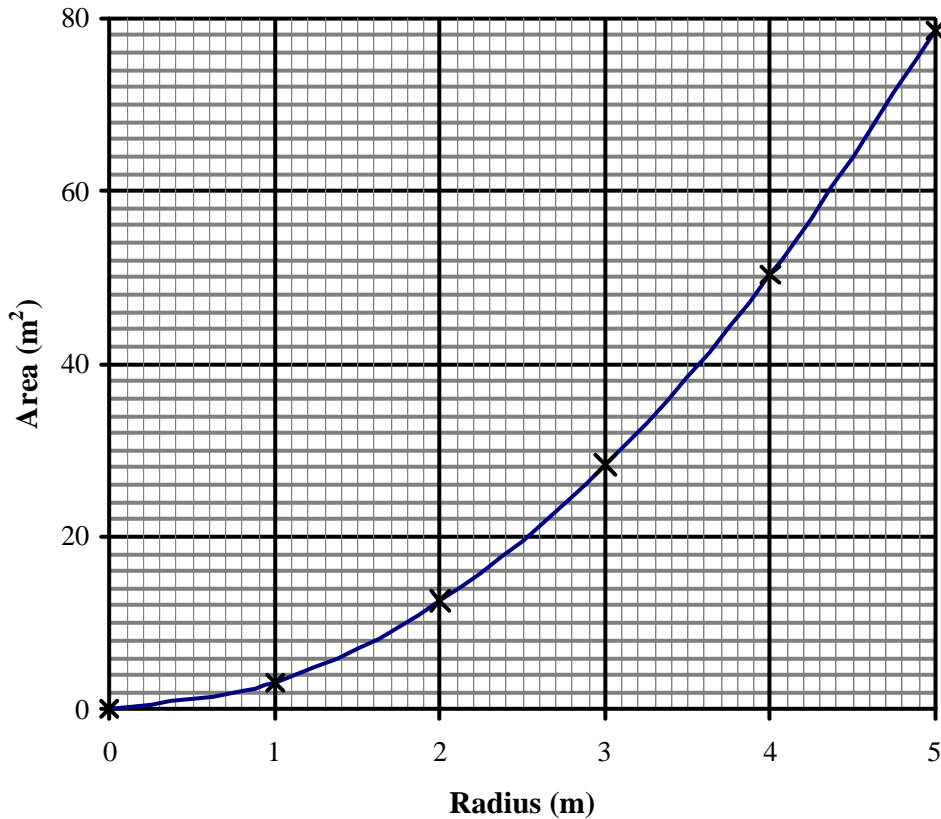
The formula $A = \pi r^2$ gives the area of a circle with radius r . Substituting values of r gives the areas shown in the table which have been rounded to one decimal place.

Radius (m)	0	1	2	3	4	5
Area (m²)	0	3.1	12.6	28.3	50.3	78.5

These values have been used to draw the graph shown below.

All pairs of values that fit the formula, give points that lie on this curve.

Area of Circles



You can use the graph to estimate the area of a circle from its radius or estimate the radius that will give a particular area. Complete the following:

If the radius is 2.5m, the area is m². If the radius is 4.3m, the area is m².

If the area is 30 m^2 , the radius is m. If the area is 66 m^2 , the radius is m.

Check your answers using the formula $A = \boldsymbol{pr}^2$



Graphs can also be drawn from measured values or a collection of data.

When points are plotted using data, errors (of measurement etc) may cause some scatter of the points about a best fit curve.

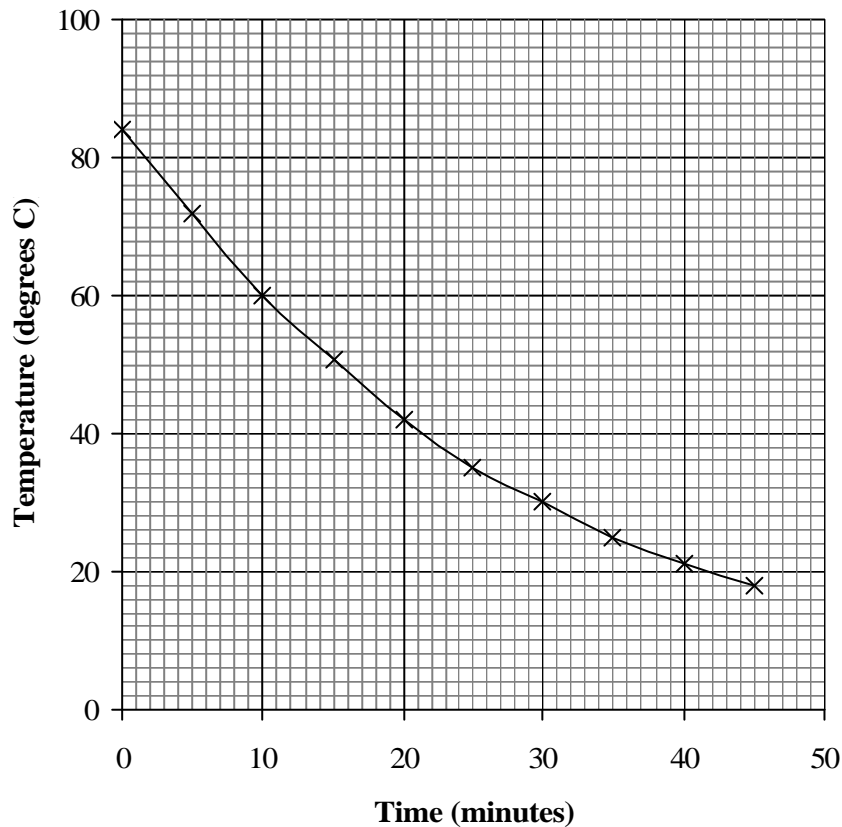
Example – Temperature of a Cup of Coffee

The temperature of a cup of coffee is measured every 5 minutes as it cools.

The results are given in the table (to the nearest degree) and shown on the graph.

Time (minutes)	0	5	10	15	20	25	30	35	40	45
Temperature (°C)	84	72	60	51	42	35	30	25	21	18

Cooling Cup of Coffee



Use the graph to complete the following:

After 12 minutes the temperature was approximately°C

After 38 minutes the temperature was approximately°C

The temperature fell to 70°C after approximately minutes

The temperature fell to 32°C after approximately minutes

What do you think will happen to the temperature after an 1 hour has passed?



.....

.....



Drawing graphs – Handy Hints

- Draw up a table of values (using data or values calculated from a formula).
- Use the **horizontal** axis for the **independent** variable and the **vertical** axis for the **dependent** variable.
e.g. the *area of circle* (dependent) depends on its *radius* (independent).
e.g. the *temperature of the cup of coffee* (dependent) depends on the *time* for which it has been cooling (independent).
- Choose suitable scales for the axes, covering the range of values in your table.
Aim to draw the graph as **large** as possible whilst **using a scale that is easy to read**.
Usually scales such as 2 cm representing 1, 2, 5, 10, 20, 50, 100, 200, 500,..... are best.
- Plot the points and draw the best fit curve/straight line through them.
- **Label** your graph fully, including a **title** and **units** (if there are any).

Some to try

College Evacuation

There were 660 people in a college at the start of a practice evacuation.

The following data was recorded during the evacuation:

Time after fire bell (t min)	0	2	4	6	8	10	12
Number of people evacuated (N)	0	340	510	590	630	650	660

- Plot a graph of N against t . (This means with N on the vertical axis and t on the horizontal axis.)
- Use the graph to estimate the following:
 - the number of people evacuated in the first 5 minutes
 - the time taken to evacuate the first 400 people
 - the time taken to evacuate 70% of the people.

Stopping Distances

According to the Highway Code, the typical stopping distances for cars travelling at different speeds are as given in the table below:

Speed (v mph)	0	20	30	40	50	60	70
Stopping distance (d metres)	0	12	23	36	53	73	96

- Plot a graph of d against v .
- Use the graph to estimate the stopping distance for a car travelling at:
 - 35 mph
 - 52 mph
 - 66 mph



- c) Use the graph to estimate how fast a car was travelling if it stops in:
- (i) 20 metres
 - (ii) 42 metres
 - (iii) 90 metres



Tides

The depth of water was measured at a coastal station every 2 hours between midnight and midday. The table gives the results.

Time after midnight (t hours)	0	2	4	6	8	10	12
Water depth (d metres)	7.77	4.96	2.05	0.37	1.94	4.72	7.53

- a) Plot a graph of d against t . Assume that low tide occurs at 6 am.
- b) Use the graph to estimate the depth of the water at the following times:
(i) 3 am (ii) 4:30 am (iii) 8:30 am (iv) 10:40 am
- c) Use the graph to estimate the times at which the depth of water was:
(i) 6 m (ii) 3 m (iii) 1.5 m (iv) 4.2 m

Surface area of a sphere

The surface area of a sphere of radius r is given by the formula $S = 4\pi r^2$

- a) Copy and complete the following table:

Radius (r cm)	0	2	4	6	8	10
Surface area (S cm ²)						

- b) Plot a graph of S against r .
- c) Use the graph to estimate the surface area of a sphere of radius:
(i) 3 cm (ii) 4.5 cm (iii) 6.8 cm (iv) 9.2 cm
- d) Use the graph to estimate the radius of a sphere whose surface area is:
(i) 400 cm² (ii) 1000 cm² (iii) 660 cm² (iv) 750 cm²

Volume of a sphere

5 The volume of a sphere of radius r is given by the formula $V = \frac{4}{3}\pi r^3$

- a) Copy and complete the following table:

Radius (r cm)	0	1	2	3	4	5
Volume (V cm ³)						

- b) Plot a graph of V against r .
- c) Use the graph to estimate the volume of a sphere of radius:
(i) 1.7 cm (ii) 2.4 cm (iii) 3.7 cm (iv) 4.6 cm



- d) Use the graph to estimate the radius of a sphere whose volume is:
(i) 100 cm^3 (ii) 250 cm^3 (iii) 360 cm^3 (iv) 475 cm^3



Teacher Notes

Unit Intermediate Level, Using algebra, functions and graphs

Skills used in this activity:

- Drawing non-linear graphs from a formula or data
- Reading values from non-linear graphs

Notes on Activity

Both of the examples on pages 1 and 2 and the handy hints for drawing graphs are included in the Powerpoint presentation of the same name which can be used to introduce this work.

The answers to the questions, including the graphs, are given below.

Answers

Page 1 Area of a circle

If the radius is 2.5m, the area is **20 m²** (to 2sf) If the radius is 4.3m, the area is **58 m²** (to 2sf)

If the area is 30 m², the radius is **3.1 m** (to 2sf) If the area is 66 m², the radius is **4.6 m** (to 2sf)

Page 2 Temperature of a cup of coffee

After 12 minutes the temperature was approximately **56 °C** (nearest °C)

After 38 minutes the temperature was approximately **22 °C** (nearest °C)

The temperature fell to 70 °C after approximately **6** minutes (nearest minute)

The temperature fell to 32 °C after approximately **28** minutes (nearest minute)

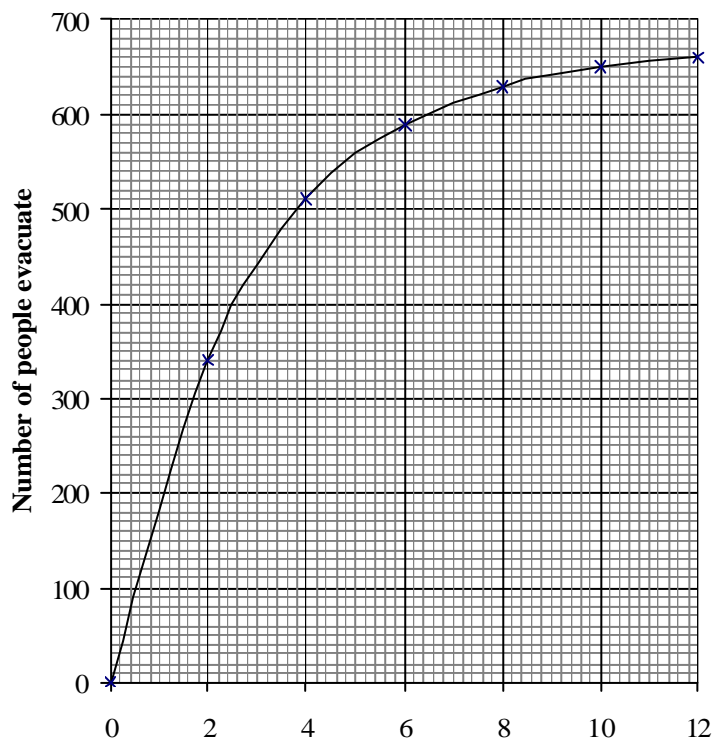
After an 1 hour the temperature will fall very slowly as the temperature of the cup of coffee approaches room temperature.

Pages 3 – 4 Some to try

College Evacuation

a) **Evacuation during a Fire Drill**

- b) (i) 560
(ii) 2.5 minutes (2 minutes 30 seconds)
(iii) 3.2 minutes (3 minutes 12 seconds)



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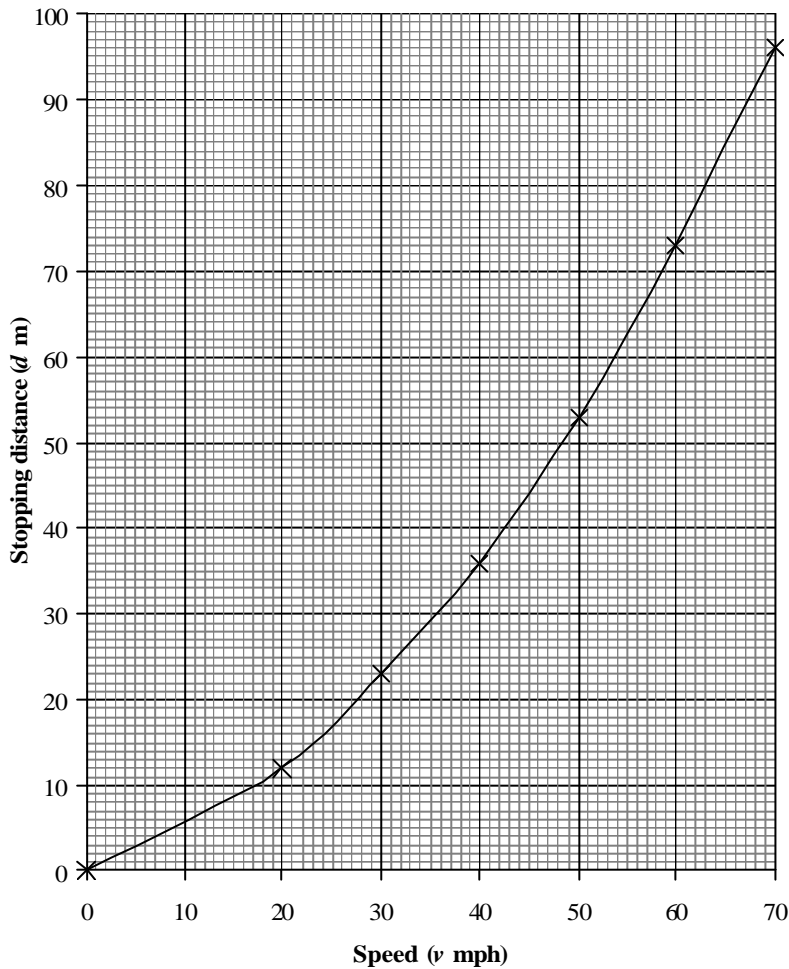
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Stopping Distances

a)

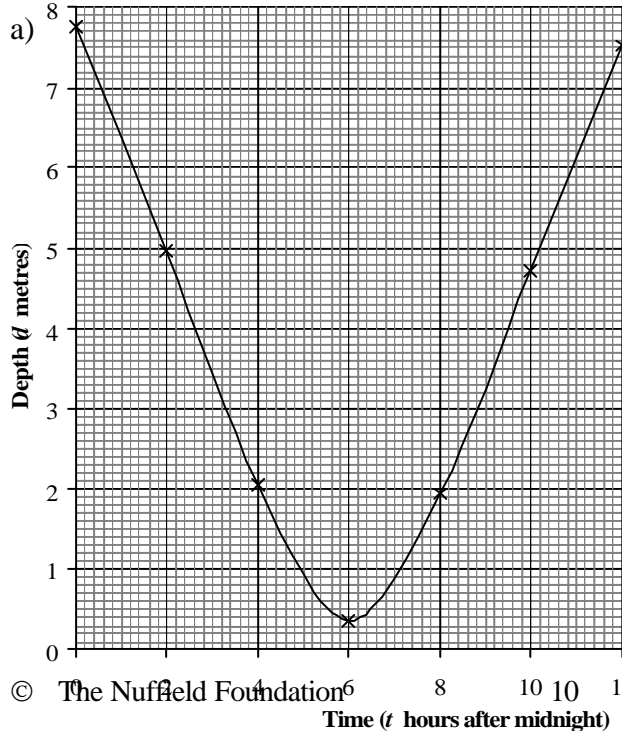
Relationship between stopping distance and speed



- b) (i) 29 metres
 (ii) 57 metres
 (iii) 87 metres
 (to nearest metre)

- c) (i) 27 mph
 (ii) 44 mph
 (iii) 67 mph
 (to nearest mph)

Tides Water depth from midnight to noon



- b) (i) 3.4 metres
 (ii) 1.5 metres
 (iii) 2.6 metres
 (iv) 5.7 metres
 (to nearest 0.1 metres)
- c) (i) 1.3 hours after midnight (01:20)
 and 10.9 hours after midnight (10:55)
 (ii) 3.3 hours after midnight (03:20)
 and 8.6 hours after midnight (08:35)
 (iii) 4.5 hours after midnight (04:30)
 and 7.6 hours after midnight (07:35)
 (iv) 2.5 hours after midnight (02:30)
 and 9.6 hours after midnight (09:35)
 (to nearest 0.1 hours or 5 minutes)



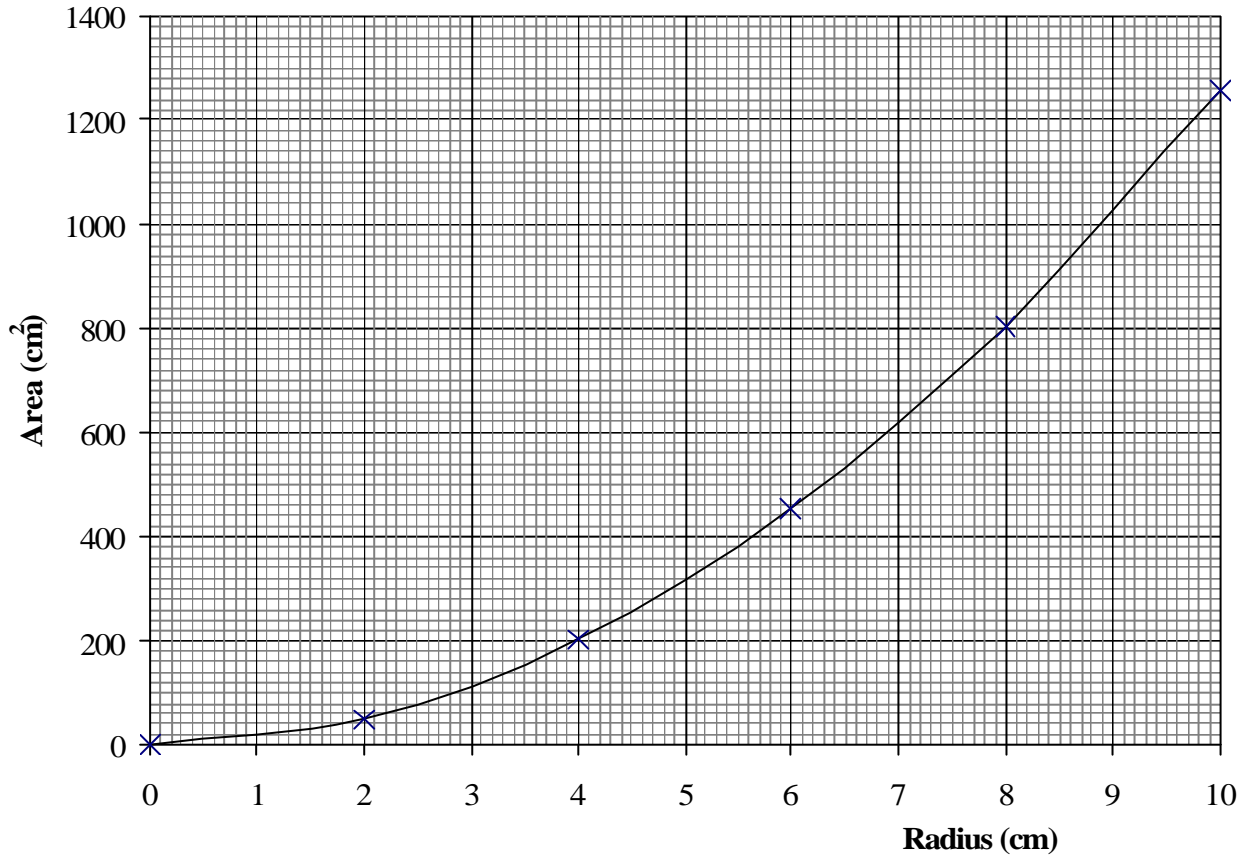
Surface area of a sphere

a)

Radius (r cm)	0	2	4	6	8	10
Surface area (S cm ²)	0	50	201	452	804	1257

b)

Surface Area of Spheres



c) (i) 110 cm² (to nearest 10 cm²) (ii) 260 cm² (iii) 580 cm² (iv) 1060 cm²

d) (i) 5.6 cm (to nearest 0.1 cm) (ii) 8.9 cm (iii) 7.2 cm (iv) 7.7 cm



Volume of a sphere

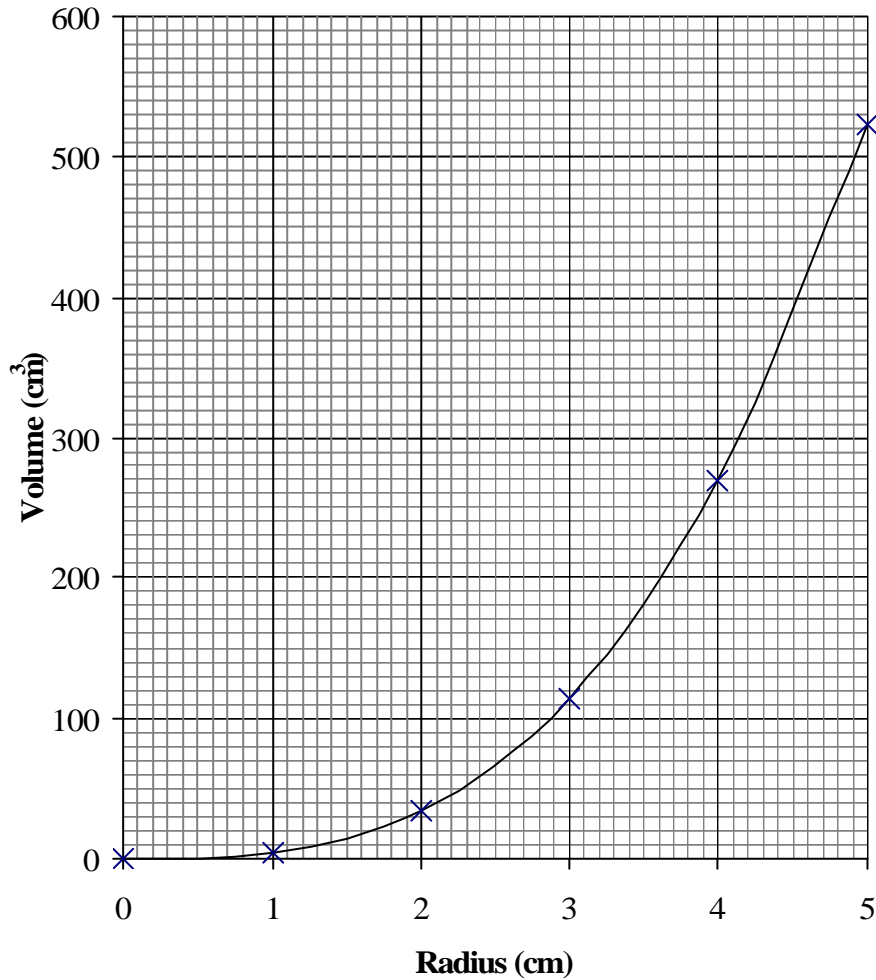
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a)

Radius (r cm)	0	1	2	3	4	5
Volume (V cm ³)	0	4	34	113	268	526

b)

Volume of Spheres



c) (i) 20 cm³ (ii) 60 cm³ (iii) 210 cm³ (iv) 420 cm³
(to nearest 10 cm³)

d) (i) 2.9 cm (ii) 3.9 cm (iii) 4.4 cm (iv) 4.8 cm
(to nearest 0.1 cm)

